

Optimal Power Flow (OPF)

In the optimal flow problem, the generation and transmission of electric energy is optimised, taking into account the active and reactive power generation limits, demand requirements, bus voltage limits, and network flow limits. Due to the non-linear power flow constraints, OPF is formulated as a non-convex optimisation problem that is generally difficult to solve.

The problem was first formulated in 1962 and a large number of optimization algorithms and relaxations have been proposed since then. A popular approximation, the directed-current optimal power flow (DCOPF), is based on the linear programming problem that is obtained through the linearisation of the power flow equations. While DCOPF is useful in a wide variety of applications, a solution of DCOPF may not satisfy the non-linear power flow equations and hence the resulting solution may be infeasible and may be of limited utility. Other types of algorithms that were studied for the OPF include Newton Raphson, quadratic programming, nonlinear programming, Lagrangian relaxation, interior point methods, genetic algorithms, and other heuristics. Although some of these algorithms can handle large-scale networks most of them can only compute a local optimal usually without assurance on the quality of the solution. That is because most of the algorithms rely on the Karush-Kuhn-Tucker necessary conditions of optimality, which cannot even guarantee a locally optimal solution, in the non-convex problem. Alternatively, the OPF can be formulated as a quadratically constrained quadratic program. There, convex relaxations within second-order cone (SOCP) programming and semidefinite programming (SDP) can be applied. In contrast to the other proposed approaches, convex relaxations make it possible to check if a solution is globally optimal. If the solution is not optimal, the relaxations provide a lower bound and hence a bound on how far any feasible solution is from optimality. In particular, Bai et al. proposed a semidefinite programming relaxation for the ACOPF for general networks, which makes it possible to find globally optimal solutions for several well-known instances. More recently, the moments and sum-of-square decomposition have been used to build hierarchies of improving SDP relaxations for a polynomial programming formulation of ACOPF. To overcome the computational complexity of using SDP and polynomial programming, graph sparsity has been exploited to simplify the SDP relaxation of the OPF.

To further improve the scalability of SDP relaxations, ADMM-based computation can be used to solve sparse, large-scale SDPs. Alternatively, cheaper hierarchies are being investigated based on LP and SOCP relaxations. In the future, a combination of hierarchies mixing constraints from different cones may be envisioned.

Another issue to address is development of techniques to certify infeasibility of optimal power flow instances.

From an industrial point of view, dealing with incomplete data is one of the issues models and tools have to address. Aggregations of industrial data may lead to physically non-meaningful models, since some section of the power network are not represented in the data.

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Software

OPF specific software

1) PowerTools

PowerTools (<http://hhijazi.github.io/PowerTools/>) is a mathematical programming library for Power Systems optimization. PowerTools is an open source c++ mathematical modeling platform linking to state-of-the-art optimization solvers. It implements problems such as Optimal Power Flow, and Optimal Transmission Switching, while benefiting from cutting-edge research results. These include smart on/off constraints modeling, efficient bound propagation, domain-specific cut generation techniques and tight convexification procedures providing global optimality guarantees. PowerTools is still under active development, future versions will include Gas expansion and Unit Commitment models, decomposition methods, multi-threading, and support to additional solvers such as Cplex, Bonmin and others. Note that PowerTools is already used by another open-source project for Smart Grid simulation: <http://nicta.github.io/SmartGridToolbox/>

2) Matpower

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5) Plexos

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General purpose solver applied to OPF

1) CPLEX

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2) MOSEK

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3) Xpress

Contributors:

Dr Martin Mevissen, IBM

Dr Cedric Jozz, LAAS CNRS, Toulouse

Dr Bissan Ghaddar, University of Waterloo

Dr Fabrizio Lacalandra, QuanTek